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NOTES ON DEBRIS-AIR-MAGNETIC INTERACTION

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PREFACE

In a nuclear explosion at high altitude, the distribution of the fission products from the explosion is determined by the interaction of the bomb debris in the earth's atmosphere and the earth's magnetic field. One model for this interaction is that of magnetohydrodynamics. However, the corrections for the individual ion trajectories in the magnetic field are large. The effect of these corrections on a magnetohydrodynamic model is examined here.

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SUMMARY

A set of hydrodynamic-like differential equations is derived for the motion of debris and air ions as they interact through the magnetic field. Although these equations are not solved in detail, one can deduce several conclusions:

(1) At early times, the fraction of the debris mixed with the magnetic field (and hence the intensity of the β-ray aurora) is proportional to the cube of the time;

(2) the outer debris ions are simply bent by the normal earth's magnetic field;

(3) air ions are picked up essentially when the debris has moved one air-ion Larmor radius.

NOTES ON DEBRIS-AIR-MAGNETIC INTERACTION

Let: n_e = density of electrons

n_d = density of debris ions

n_a = density of air ions

The Coulomb forces are very strong, so approximately,

$$n_e = n_d + n_a \quad . \tag{1}$$

An indication of the strength of the Coulomb forces is obtained by comparing the Debye length with the dimensions of the problem. Taking an electron temperature of 10 e.v., which is characteristic, we find the Debye length

$$\lambda_D = \frac{kT}{4\pi\, n_e\, e^2} = \frac{6 \times 10^6}{n_e} \quad . \tag{2}$$

Therefore for electron densities larger than 10^6 the Debye length is very short compared to the dimensions of the problem. Electron and ion charges cannot be separated by more than a Debye length.

The various Larmor radii, assuming a velocity of 2×10^8 cm/sec and B = 1/3 gauss are:

$$\begin{aligned} &\text{electrons } \frac{mvc}{eB} = 33 \text{ cm} \\ &\text{air ion } (O^+) = 10 \text{ km} \\ &\text{debris ion } (M = 100) = 60 \text{ km} \quad . \end{aligned} \tag{3}$$

Since the electron Larmor radius is very short, the approximation of magnetohydrodynamics is very good for the electrons. This means that the electrons and magnetic field are stuck together. The ions, however, in the earlier stages of the explosion, are almost unaffected by the magnetic field. We therefore, get the following picture, roughly, in the earlier stages.

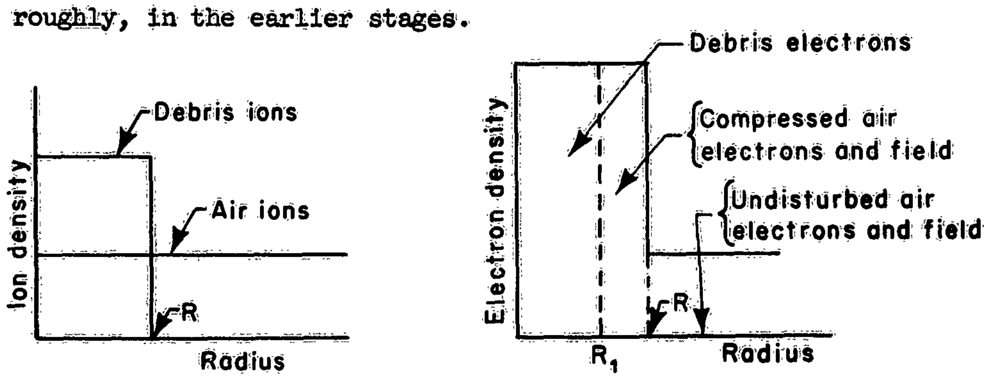

Fig. I

(See Fig 1.) At some time, the radius of the debris ions, which are essentially coasting, is R. The air ions have not yet been picked up because R is considerably less than an air ion Larmor radius. The electrons, which have very little rigidity, accommodate the total ion density. The magnetic field, which is very weak, is simply carried along by the air electrons. The radius R_1 represents the boundary between a spherical <u>bubble</u>, containing debris electrons and no magnetic field, and a spherical <u>shell</u>, containing compressed air electrons and magnetic field. The magnetic field pattern is as shown in Fig 2.

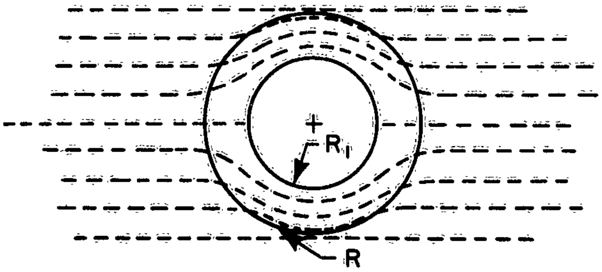

Fig. 2

Two important consequences of this picture are as follows. First, a fraction of the debris is mixed with the magnetic field early, and β-rays born in the shell can escape immediately (β-rays born in the bubble are trapped). Secondly, the radius of the β-rays aurora will be exactly equal to the radius of the debris, since the magnetic field is undisturbed outside the debris.

The fraction of the debris mixed with the magnetic field is determined by the ratio of the radii R_1 and R. R_1 is determined by the requirement of neutrality. Let the total number of debris ions (and of electrons) be N_o. Then Eq. (1), applied to the region inside R_1, becomes

$$\frac{N_o}{\frac{4\pi}{3} R_1^3} = \frac{N_o}{\frac{4\pi}{3} R^3} + n_a$$

Solving, we find

$$\left(\frac{R_1}{R}\right)^3 = \frac{1}{1 + \frac{4\pi}{3} \frac{n_a R^3}{N_o}} \tag{4}$$

(In this analysis we have assumed that the debris density and the air density are uniform.) The fraction of the debris mixed with the magnetic field is

$$F.M. = 1 - \left(\frac{R_1}{R}\right)^3 = \frac{\frac{4\pi}{3}\frac{n_a R^3}{N_o}}{1 + \frac{4\pi}{3}\frac{n_a R^3}{N_o}} \tag{5}$$

Since R is proportional to the time, it is clear that F.M., and the total intensity of the β-ray aurora, are roughly proportional to the cube of the time at early times. (The β-ray decay rate is assumed constant up to one second.)

Numerically, let us take $N_o = 10^{28}$ ions and $n_a \approx 10^8$ ions/cm^3. Thus, for small R,

$$F.M. \approx 0.04 \left(\frac{R}{10\text{km}}\right)^3 \tag{6}$$

Actually, n_a is not a constant, since most of the air ions are made by x-rays, so this result should not be taken too seriously.

Eventually, both debris and air ions begin to be deflected on account of their motion relative to the magnetic field--that is, they begin to be stuck in the magnetic field. This happens to the air ions, for example, when the magnetic field has been moved a fraction of 10 km, the air ion Larmor radius.

Let us see whether the magnetic field can supply enough stress to provide momentum for the picked up air ions. The magnetic

pressure is

$$\frac{B^2}{8\pi} = (\tfrac{1}{3})^2 /8\pi = 4 \times 10^{-3} \text{ erg/cm}^3 \tag{7}$$

the stress needed to pick up air ions is

$$n_a M_a v^2 = (10^8)(2.5 \times 10^{-23})(4 \times 10^{16})$$

$$= 10^2 \text{ erg/cm}^3 \tag{8}$$

Clearly the magnetic pressure is completely inadequate to pick up the air ions. On the other hand, the ions are clearly bent by the magnetic field, and we have a paradox. The explanation of the paradox is as follows: When the ions are deflected sideways, they make an electric current $\vec{J}_1$, and the force $\vec{J}_1 \times \vec{B}$ is the force needed to pick up the air ions. An exactly equal but opposite current is induced in the electrons, so that the magnetic field simply passes on the force to the electrons. The electrons also cannot themselves supply the needed stress, but, after a small separation of charge, a radial electric field is set up, and through it the electrons pass on the force to the debris ions, which have most of the momentum.

Let us now express these ideas quantitatively. Let $\vec{J}_e, \vec{J}_a, \vec{J}_d$ be the electric currents due to electrons, air ions, and debris ions, respectively. Then the statement that the magnetic field supplies negligible stresses means that

$$\vec{J}_e + \vec{J}_a + \vec{J}_d \approx 0 \tag{9}$$

so that $\vec{J} \times \vec{B} \approx 0$. Note also that this equation is consistent with the maintenance of neutrality. Next, the statement that the electrons supply negligible stress means that

$$\vec{J}_e \times \vec{B} - e\, n_e \vec{E} = 0 \tag{10}$$

where the charge e is taken positive and $\vec{E}$ is the electric field. Combining (9) and (10) we may solve for $\vec{E}$ in terms of the ion currents,

$$e\vec{E} = -\frac{1}{n_e}(\vec{J}_a + \vec{J}_d) \times \vec{B} \tag{11}$$

The equations of motion of the two ion streams are

$$\begin{aligned} n_a m_a \frac{d\vec{v}_a}{dt} &= n_a\, e\vec{E} + \vec{J}_a \times \vec{B} \\ n_d m_d \frac{dv_d}{dt} &= n_d\, e\vec{E} + \vec{J}_d \times \vec{B} \end{aligned} \tag{12}$$

where m_a and m_d are ion masses, $\vec{v}_a$ and $\vec{v}_d$ are the velocities. Using Eq. (11) to eliminate $\vec{E}$, using Eq. (1), and the fact that

$$\vec{J}_a = n_a\, e \frac{\vec{v}_a}{c} \quad , \quad \vec{J}_d = n_d\, e\, \frac{\vec{v}_d}{c} \tag{13}$$

the equations of motion become

$$\frac{d\vec{v}_a}{dt} = \frac{e}{m_a c}\ \frac{n_d}{n_e}(\vec{v}_a - \vec{v}_d) \times \vec{B} \tag{14}$$

$$\frac{d\vec{v}_d}{dt} = \frac{e}{m_d c} \frac{n_a}{n_e} (\vec{v}_d - \vec{v}_a) \times \vec{B} \tag{15}$$

From these equations it follows that the total ion momentum is constant:

$$n_a m_a \frac{d\vec{v}_a}{dt} + n_d m_d \frac{d\vec{v}_d}{dt} = 0$$

This is consistent with our neglecting the stresses of the magnetic field and the electrons.

Equations (14) and (15) govern the time development of the ion stream velocities. It must be remembered that the $\frac{d}{dt}$ in these equations are total derivatives.

We shall not solve these equations here. The solution may be a job for a computing machine. However it is useful to consider the motion of the outer most debris mass point. For this mass point

$$\frac{\vec{B}}{n_e} = \frac{\vec{B}_o}{n_a}$$

where B_o is the initial magnetic field. Therefore, for this mass point,

$$\frac{d\vec{v}_d}{dt} = \frac{e}{m_d c} \vec{v}_d \times \vec{B}_o ,$$

since $\vec{v}_a$ has not yet changed from zero at this point. Thus the outer most ions act as if they were simply bent by the original magnetic field. However, they may be soon overtaken by debris ions from behind, and cease to be the outermost debris.

It is also instructive to examine the solution of the air ion

equation of motion for small times when $\vec{v}_a \ll \vec{v}_d$. In this case $\frac{n_d}{n_e} \approx 1$. Integrating Eq. (14) once, neglecting v_a on the right, to first order (t is the time)

$$\vec{v}_a^{(1)} = - \frac{et}{m_a c} \vec{v}_d \times \vec{B} \quad .$$

Putting this result back in Eq. (14) on the right, one finds a second order correction to $\vec{v}_a$,

$$\vec{v}_a^{(2)} = \frac{1}{2} \left(\frac{eBt}{m_a c}\right)^2 \vec{v}_d \quad .$$

This shows that the air ions begin to be picked up substantially in $\frac{1}{2\pi}$ Larmor periods of the air ions in the compressed magnetic field.